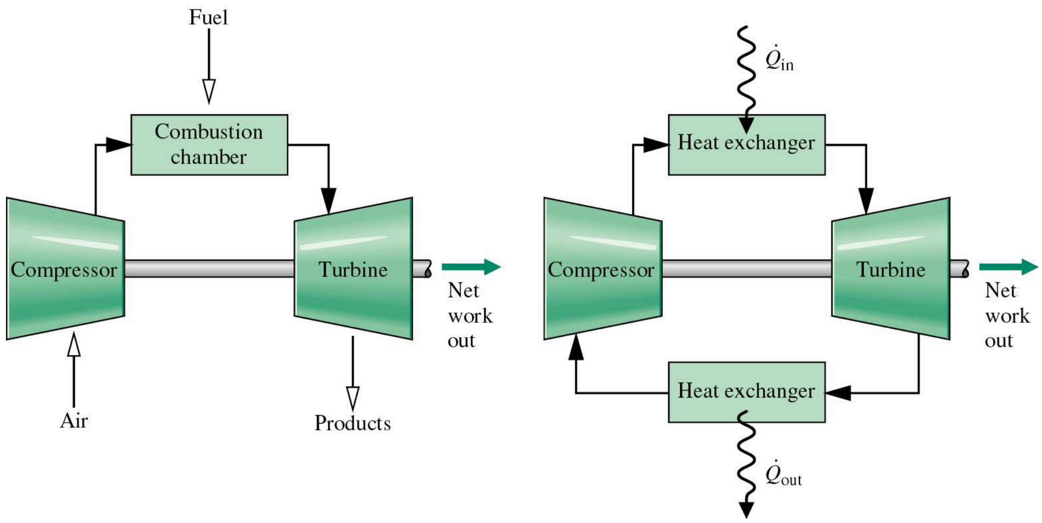
Analysis of Thermodynamic Cycles

Applications For Modeling Turbine Systems

Gas turbine systems operate on what is known as a Brayton Cycle. This cycle in its most basic form consists of three or four stages – depending on if the cycle is open (air is drawn into the system and vented after the combustion / energy extraction) or closed (there is a single gas flow that continuously circulates through the turbine system) – as shown below:



Standard Brayton Cycles.
Image taken from <http://www.chrisbence.com/>

Typical gas turbine power plants operate in an open 3-stage cycle with air transitioning between four primary states throughout the process. As a result, these four states can be analyzed to determine the expected energy produced / lost in the system and overall system efficiency.

Basic Thermodynamic Analysis (Ideal)

Process:

Isentropic Process (1→2) - Compression in the compressor at constant entropy
Isobaric Process (2→3) - Combustion of fuel / air mixture at constant pressure
Isentropic Process (3→4) - Expansion in the turbine at constant entropy
Isobaric Process (4→1) - Rejection of heat / products at constant pressure

Calculations:

The standard energy balance equation for a steady-state process is given as

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = (h_{outlet} - h_{inlet})$$

$$q_{in} = h_3 - h_2 = C_p(T_3 - T_2)$$

$$q_{out} = h_4 - h_1 = C_p(T_4 - T_1)$$

with heat transfer (*q*), work (*w*), and enthalpy (*h*). The values for enthalpy can be calculated under the simplifying conditions – known as a cold air standard cycle – where the specific heats for gases are constant across a range of temperatures. Determining the enthalpy at each state (in these simplified conditions) requires knowledge of the state's temperature. These can be determined from the state's pressure and entropy as follows. Assuming that Process 1-2 & Process 3-4 are isentropic (constant entropy) provides for:

$$s_1 = s_2$$

$$s_3 = s_4$$

Assuming that Process 2-3 & Process 4-1 are isobaric provides for:

$$P_1 = P_4$$

$$P_2 = P_3$$

Combining these two facts with the following equation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\frac{K-1}{K})} = \left(\frac{P_3}{P_4}\right)^{(\frac{K-1}{K})} = \frac{T_3}{T_4}$$

allows for the temperature to be specified for all 4 states. The enthalpy (*h*) of the gas can then be calculated for the different states and used in conjunction with the energy balance equation to determine the net work produced by the system (effectively the power that the turbine is generating) from the term (w_{in} – w_{out}). The efficiency of the turbine can also be calculated as followed:

$$\eta_{Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p(T_3 - T_2)}{C_p(T_4 - T_1)}$$

$$= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Basic Thermodynamic Analysis (Actual)

Process:

Adiabatic Process (1→2) - Compression in the compressor without heat transfer
Isochoric Process (2→3) - Combustion of fuel / air mixture at constant volume
Adiabatic Process (3→4) - Expansion in the turbine without heat transfer
Isochoric Process (4→1) - Rejection of heat / products at constant volume